Tutorial 3: Further Questions (not marked)

1) Cerium Magnesium Nitrate (CMN) is a spin-1/2 paramagnetic salt that is used in magnetic cooling.

i) The magnetic heat capacity of the spin-1/2 salt at a temperature T and a magnetic field B is given by

$$C = Nk_B x^2 \mathrm{sech}^2 x\,,$$

where $x = \dfrac{\mu_B B}{k_B T}$. Show that the high temperature approximation is given by

$$C_a = Nk_B x^2\,.$$

For this to be valid, how should $k_B T$ compare with $\mu_B B$?

ii) Show that the approximate heat capacity is always greater than the exact one.

iii) For a magnetic field of 1 T, at what temperature does the high temperature approximation for the heat capacity deviate by 5% from the exact results.

iv) 75% of the paramagnetic ions are polarised with their spin in one direction, and the rest in the opposite direction. Show that

$$e^{-2x} = 0.75.$$

v) To what temperature does one have to refrigerate the salt so that this would happen.

A sample of CMN is initially at a magnetic field of 2 T and a temperature of 1 K.

vi) It is demagnetised to 0.003 T. During demagnetisation, the temperature is proportional to field. Calculate the final temperature.

The magnetic energy in a spin ½ salt is given by

$$U = -N\mu_B B \tanh x\,,$$

where N is the number of particles.

vii) Find the magnetic energy in one mole of the salt, at a final field of 3 mT and final temperature of 1 mK.

viii) Find the limiting energy at high temperature.

ix) The cooling power of the CMN is the heat that it would absorb when it warms up, which is converted to the energy. Calculate the cooling power of one mole of CMN.

The magnetic entropy in a spin-1/2 salt is given by

$$S = Nk_B \ln(2\cosh x) - Nk_B x \tanh x\,.$$

x) State the entropy at a temperature of 1 K and zero field for one mole of the salt. Calculate the entropy when the field is increased to 2 T.

xi) For a reversible change, the heat absorbed is related to the entropy change by

$$dQ = T\,dS.$$

Calculate the heat of magnetisation which has to be removed if this salt is magnetised isothermally from 0 to 2 T